5.2 Trigonometric Functions of Real Numbers

*(You might find it helpful to review your general knowledge of functions 2.1-2.6_*

Definition of the Trigonometric Functions

Let t be any real number and let $P(x, y)$ be the terminal point on the unit circle determined by t. We define

$$\sin t = y \qquad \cos t = x \qquad \tan t = \frac{y}{x} \quad (x \neq 0)$$

$$\csc t = \frac{1}{y} \quad (y \neq 0) \qquad \sec t = \frac{1}{x} \quad (x \neq 0) \qquad \cot t = \frac{x}{y} \quad (y \neq 0)$$

Notation: can be written $\sin t$ or $\sin(t)$

Examples Quadrant 1

$\sin(\pi/6) =$ _____ $\csc(\pi/6) =$ _____

$t = \pi/6$ $\cos(\pi/6) =$ _____ $\sec(\pi/6) =$ _____

$\tan(\pi/6) =$ _____ $\cot(\pi/6) =$ _____

$\sin(\pi/4) =$ _____ $\csc(\pi/4) =$ _____

$t = \pi/4$ $\cos(\pi/4) =$ _____ $\sec(\pi/4) =$ _____

$\tan(\pi/4) =$ _____ $\cot(\pi/4) =$ _____

$\sin(\pi/3) =$ _____ $\csc(\pi/3) =$ _____

$t = \pi/3$ $\cos(\pi/3) =$ _____ $\sec(\pi/3) =$ _____

$\tan(\pi/3) =$ _____ $\cot(\pi/3) =$ _____

Reciprocal Identities

$\sec(\pi/3) =$ _____ $\csc(\pi/4) =$ _____ $\cot(\pi/6) =$ _____

Examples Quadrantal Numbers (Angles)

$\sin(0) =$ _____ $\csc(0) =$ _____

$\cos(0) =$ _____ $\sec(0) =$ _____

$\tan(0) =$ _____ $\cot(0) =$ _____

$\sin(\pi/2) =$ _____ $\csc(\pi/2) =$ _____

$\cos(\pi/2) =$ _____ $\sec(\pi/2) =$ _____

$\tan(\pi/2) =$ _____ $\cot(\pi/2) =$ _____

$\sin(\pi) =$ _____ $\csc(\pi) =$ _____

$\cos(\pi) =$ _____ $\sec(\pi) =$ _____

$\tan(\pi) =$ _____ $\cot(\pi) =$ _____

$\sin(3\pi/2) =$ _____ $\csc(3\pi/2) =$ _____

$\cos(3\pi/2) =$ _____ $\sec(3\pi/2) =$ _____

$\tan(3\pi/2) =$ _____ $\cot(3\pi/2) =$ _____

Domain of Trigonometric Functions

Examples: Evaluating Trig Functions in Other Quadrants

$\sin(\pi/3) =$ _____ $\sin(2\pi/3) =$ _____

$\sin(4\pi/3) =$ _____ $\sin(5\pi/3) =$ _____

Notice:

$\cos(\pi/6) =$ _____ $\cos(5\pi/6) =$ _____

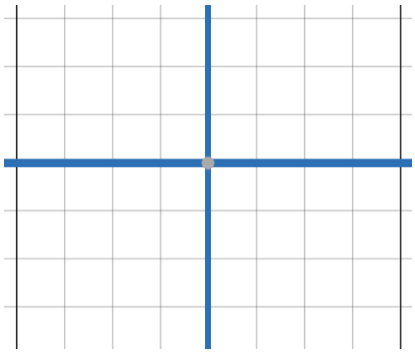
$\cos(7\pi/6) =$ _____ $\cos(11\pi/6) =$ _____

$\tan(\pi/4) =$ _____ $\tan(3\pi/4) =$ _____

$\tan(5\pi/4) =$ _____ $\tan(7\pi/4) =$ _____

So Trig Functions having the same reference number (angle) have the same absolute value, but may differ in sign depending on the quadrant.

Signs:



Example: Quadrant of a Terminal Point

In what quadrant is $\sin t < 0$ and $\sec t > 0$_____

Examples: Evaluating Trig Functions Using only Reference Numbers and Quadrant Signs

$\cos(11\pi/6) =$ _____ $\sin(4\pi/3) =$ _____

$\tan(5\pi/6) =$ _____ $\sec(5\pi/4) =$ _____

$\csc(5\pi/3) =$ _____ $\cot(7\pi/4) =$ _____

Examples: Relationship Between Trig Functions at t and at $-t$

$\cos(\pi/4) =$ _____ $\cos(-\pi/4) =$ _____

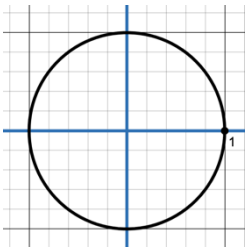
$\sin(\pi/6) =$ _____ $\sin(-\pi/6) =$ _____

$\tan(\pi/3) =$ _____ $\tan(-\pi/3) =$ _____

Even Odd Properties of Trigonometric Functions

Recall: Even Function Odd Function

_____



$\sin(-t) =$ _____ $\cos(-t) =$ _____ $\tan(-t) =$ _____

Example: Using Even/Odd Properties

$\cos(-2\pi/3) =$ _____ $\sin(-7\pi/4) =$ _____ $\tan(-\pi/6) =$ _____

Estimating Trig Values With a Calculator –

$\cos(\pi/8) =$ _____ $\sin(-5\pi/12) =$ _____ $\tan(3) =$ _____

$\sec(4) =$ _____ $\cot(100) =$ _____ $\csc(1.3) =$ _____

Very IMPORTANT: _____

Pythagorean Identities

Example: We use the notation $\cos^2(t)$ to mean _____

Compute

$$\cos^2(\pi/3) + \sin^2(\pi/3)$$

Pythagorean Identities		
$\cos^2(t) + \sin^2(t) =$ __	$\cos^2(\pi/3) + \sin^2(\pi/3)$	$\cos^2(\pi/3) + \sin^2(\pi/3)$

Example: Finding All Trig Values Given the Value of One of Them

Given that $\cos(t) = -\frac{3}{5}$ and t is in Quadrant III, find the values of the other 5 trig functions as t

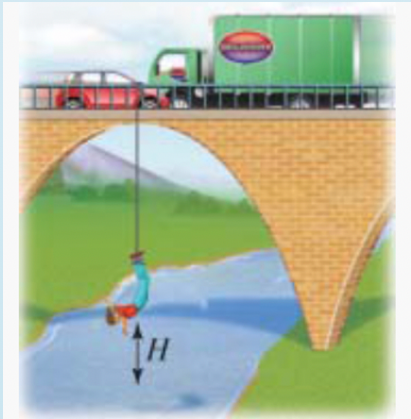
Example: Writing One Trig Function in Terms of Another

Write $\tan(t)$ in terms of $\sin(t)$ for t in Quadrant III

Applications

82. **Bungee Jumping** A bungee jumper plummets from a high bridge to the river below and then bounces back over and over again. At time t seconds after her jump, her height H (in meters) above the river is given by $H(t) = 100 + 75e^{-t/20}\cos\left(\frac{\pi}{4}t\right)$. Find her height at the times indicated in the table.

t	$H(t)$
0	
1	
2	
4	
6	
8	
12	



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